# 2 Chapter 4 Appendix

## 2.1 A Theory of Religious Identity, Conflict and Cooperation

### 2.1.1 Endowments and Technologies

Consider three countries at time zero when each country $i$, $i =$**A**, **B**, **C**, is endowed with an identical amount of $z$. The intra-temporal output of country $i$ in period $t$, $y_t^i$, is produced using its endowment input $z^i$ net of lump-sum taxes $\tau_t^i$:

$$y_t^i = z^i - \tau_t^i \ . \tag{A.4.1}$$

Each country is ruled by a sovereign who has an infinite time horizon. In every period $t$, this sovereign has the power to tax his country's endowment base $z$ to raise revenue and use it to contest the ownership of the endowments of another country via military action or defend his territory against hostility from others.

If a country declares war on another, both countries' endowments becomes contestable. Country $i$ wins the war with probability $p_t^{ij}$ and country $j$ wins it with probability $1-p_t^{ij} \equiv p_t^{ji}$. The victorious country claims all of the contested endowments $2z$ as its own.

Military power depends on the military technology parameters $\beta^i$, $i =$**A**, **B**, **C**, and the total amount of resources devoted to military spending. In turn, the relative strength of the militaries decides the expected likelihood of winning a war. That is, if countries $i$ and $j$ confront each other in period $t$, then the probability of $i$ winning the conflict is

$$p_t^{ij} = \frac{\beta^i \tau_t^i}{\beta^i \tau_t^i + \beta^j \tau_{jt}} \tag{A.4.2}$$

where $i$, $j =$**A**, **B**, **C**, $i \neq j$, and $\beta^i$, $\beta^j > 0$, representing the potency of country $i$ and $j$'s military strengths. Increases in $i$'s military strength through higher military spending $\tau_t^i$ raise the likelihood that country $i$ wins the military conflict, and increases in $j$'s military strength lowers the likelihood that country $i$ can claim victory. For expositional simplicity, I assume $\beta^A \equiv 1$.

Besides other factors, the $\beta$'s are functions of whether or not two military foes subscribe to the same faith. In particular, implicit in our discussion is the notion that when two countries adhere to two different religions, their $\beta$'s are higher in conflict. As well, this notion can apply within religious faiths—albeit with less intensity and fervor—to the extent that two potential rivals share a common religion but subscribe to two different sects within it.

The sovereign in country $i$ maximizes his country's net expected discounted output over time:

$$\max_{\tau_t^{ij}} \sum_{t=0}^{\infty} \delta^t p_t^{ij} (Z_t - \tau_t^i) \tag{A.4.3}$$

where $\delta$, $0 < \delta < 1$, represents the sovereigns' time discount factor, $Z_t \in \{z, 2z, 3z\}$ denotes the endowment base of country $i$ at time $t$, and where, in every period $t$, the state budget needs to be balanced:

$$\tau_t^i \leq Z_t \ . \tag{A.4.4}$$

### 2.1.2 Three Cases

Given the geographical alignments of the three countries and the limitations of military technology expressed above, there are three equilibria we need to investigate: In one, Country **A** and **B** engage in a military conflict, while Country **C** finds it in its interest not to interfere. Then, depending on the outcome of that conflict, Country **C** engages either Country **A** or **B** subsequently. At the end of two periods, there will be one country left standing with all the resources at its disposal thereafter. Recall that Figures A.4.2 and A.4.3 above depict this case under the assumption that Country **A** prevails over **B** in the first period.

In a second scenario, Country **B** and **C** engage in a military conflict at the outset, while Country **A** sits on the sideline. In the following period, Country **A** confronts the winner of the war between Country **B** and **C**.

In the third and final scenario, peace prevails indefinitely although this does not imply that no country chooses to arm militarily.

In what follows, I will ignore two other potentially relevant equilibria in which two countries form a coalition against the remaining country and engage it militarily at time zero (i.e., Country **A** and **B** collude against Country **C** or Country **B** and **C** join forces against Country **A**). Such collaborations require commitment between colluding countries which is typically hard if not impossible to enforce. The commitment problem arises because it would be in the interest of the stronger partner of any alliance to renege on its promise not to attack its weaker collaborator after their joint foe is defeated or, at the very least, disregard the agreed upon division of the spoils of victory. This problem could be overcome only if the stronger agent can ex ante commit to an agreement which would be in effect after the enemy if defeated.

Returning back to the three cases that we shall investigate:

**(a)** Working our way backward, we begin at $t = 1$ when Country **C** takes on the winner of the conflict between **A** and **B**. Let $v$, $v =$ **A**, **B**, represent the victor of the first conflict at $t = 0$. At $t = 1$, countries **C** and $v$ respectively solve the following problems:

$$\max_{\tau_1^C} \left( z - \tau_1^C \; + \; 3zp_1^{Cv} \sum_{t=2}^{\infty} \delta^{t-1} \right) \tag{A.4.5}$$

and

$$\max_{\tau_1^v} \left( 2z - \tau_1^v \; + \; 3z(1 - p_1^{Cv}) \sum_{t=2}^{\infty} \delta^{t-1} \right) \tag{A.4.6}$$

subject to equations (A.4.1), (A.4.2) and (A.4.4).

According to (A.4.5) and (A.4.6), country **C** enters $t = 1$ with an endowment of $z$ because it has not engaged in conflict at $t = 0$, whereas country $v$ begins $t = 1$ with an endowment of $2z$ because it has captured the endowment of its rival at $t = 0$. For Country **C**, the expected likelihood of winning its conflict with $v$ equals $p_1^{Cv}$, and for $v$, that likelihood is equal to $1 - p_1^{Cv}$. Whichever country wins the war at $t = 1$ claims all of the endowments, $3z$, and ensure not to face a rival at any future date $t > 2$.

In all that follows, I assume that the free parameter values are such that we get interior solutions. With that, equations (A.4.5) and (A.4.6) yield $\tau_1^C = \tau_1^v \equiv \bar{\tau}_1$ where

$$\bar{\tau}_1 = \frac{\beta^C \beta^v}{(\beta^C + \beta^v)^2} 3\Delta z \; ; \qquad \Delta \equiv \frac{\delta}{1 - \delta} \; . \tag{A.4.7}$$

The optimal amount of resources allocated to military buildup are identical for the two countries; it rises with the total endowment base, $3z$, and the combined military strengths, $\beta^3$ and $\beta^v$.

On the basis of (A.4.7), we can express the net expected value of scenario (a) to countries **C** and $v$ at time 1 respectively as follows:

$$V_1^C = \left[1 + \left(\frac{\beta^C}{\beta^C + \beta^v}\right)^2 3\Delta\right] z \tag{A.4.8}$$

and

$$V_1^v = \left[2 + \left(\frac{\beta^v}{\beta^C + \beta^v}\right)^2 3\Delta\right] z \ . \tag{A.4.9}$$

According (A.4.8), the expected value to Country **C** of remaining at peace in period zero and then engaging in period one the country that emerges victorious in its conflict at time zero is an increasing function of its endowment base $z$ as well as its military conflict technology $\beta^C$, but it is a decreasing function of the potency of the military technology of its rival $\beta^v$. In analogous fashion, (A.4.9) suggests that the expected value to country $v$ of engaging its neighbor first and Country **C** next rises with $z$ and $\beta^v$ whereas it falls with $\beta^C$.

Now consider the choices made by the sovereigns of Country **A** and **B** in period 0:

$$\max_{\tau_0^i} \ \left(z - \tau_0^i \ + \ \delta p_0^{ij} V_1^i\right) \tag{A.4.10}$$

subject to equations (A.4.1), (A.4.2), (A.4.4), (A.4.9) and where $i$, $j$ = **A**, **B**, $i \neq j$.

Solving the problem in (A.4.10) for both $i$ and $j$ yields $\bar{\tau}_0^B = \Omega \bar{\tau}_0^A$ where

$$\Omega \equiv \left[2 + \left(\frac{\beta^B}{\beta^B + \beta^C}\right)^2 3\Delta\right] \Big/ \left[2 + \left(\frac{1}{1 + \beta^C}\right)^2 3\Delta\right] > 1 \ . \tag{A.4.11}$$

As a result, we get

$$\bar{\tau}_0^A = \frac{\delta \beta^2 \Omega z}{(1 + \beta^B \Omega)^2} \left[2 + \left(\frac{1}{1 + \beta^C}\right)^2 3\Delta\right] \ , \tag{A.4.12}$$

and

$$\bar{\tau}_0^B = \frac{\delta \beta^2 \Omega z}{(1 + \beta^2 \Omega)^2} \left[2 + \left(\frac{\beta^B}{\beta^B + \beta^C}\right)^2 3\Delta\right] \ . \tag{A.4.13}$$

On the basis of (A.4.12) and (A.4.13), we can express the net expected value of scenario (a) to Country **A** and **B** at time 0 respectively as

$$ {}^{a}V_0^A = \left\{ 1 + \delta \left( \frac{1}{1+\beta^B \Omega} \right)^2 \left[ 2 + \left( \frac{1}{1+\beta^C} \right)^2 3\Delta \right] \right\} z \tag{A.4.14} $$

and

$$ {}^{a}V_0^B = \left\{ 1 + \delta \left( \frac{\beta^B \Omega}{1+\beta^B \Omega} \right) \left( \frac{(\beta^B - 1)\Omega + 1}{1+\beta^B \Omega} \right) \left[ 2 + \left( \frac{\beta^B}{\beta^B + \beta^C} \right)^2 3\Delta \right] \right\} z \tag{A.4.15} $$

In terms of notation, note that the lowercase superscript $a$ to the left of the value function, $V$, denotes the latter under case (a). (A.4.14) and (A.4.15) have some of the same properties of (A.4.8) and (A.4.9): increases in the endowment base $z$ raise them and countries' own military potencies do too, but increases in their opponents' military might reduces both (A.4.14) and (A.4.15). What is different, however, is that all three countries' military technology parameters, $\beta$s, have a bearing on (A.4.14) and (A.4.15) but not on (A.4.8) and (A.4.9). At time one, the expected values are expressed conditional on survival in period zero, when both Country **A** and **B** face the prospect of fighting two wars back to back. Their appraisal of the future implicitly reflects surviving both those challenges, which in turn depend on the military technologies of all three players.

For this equilibrium to be stable, Country **C** ought to find it optimal to decide not to attack Country **B** at $t = 0$. Moreover, if Country **B** would have to engage **A** at the outset and its optimal for Country **C** to delay an attack on **B**, then it would be optimal for $i = \mathbf{C}$ not to invest any resources to its military.

Using equation (A.4.8), we can derive the expected value to Country **C** of remaining idle at $t = 0$ as

$$ {}^{a}V_0^C = \left\{ 1 + \delta + 3\delta\Delta \left[ \left( \frac{1}{1+\beta^B \Omega} \right) \left( \frac{\beta^C}{1+\beta^C} \right)^2 + \left( \frac{\beta^B \Omega}{1+\beta^B \Omega} \right) \left( \frac{\beta^C}{\beta^B + \beta^C} \right)^2 \right] \right\} z \tag{A.4.16} $$

By comparing (A.4.16) with the expected value to $i = \mathbf{C}$ of engaging $i = \mathbf{B}$ immediately, we can determine whether or not case (a) is sustainable as an equilibrium. To that end, consider this:

$$\frac{\partial({}^{a}V_0^C)}{\partial \beta^B} = - \left\{ \begin{array}{c} \frac{\Omega}{(1+\beta^B\Omega)^2}\left[\left(\frac{\beta^C}{1+\beta^C}\right)^2 - \left(\frac{\beta^C}{\beta^B+\beta^C}\right)^2\right] \\ \\ +\left(\frac{2\beta^B\Omega}{1+\beta^B\Omega}\right)\frac{\beta^C}{(\beta^B+\beta^C)^3} \end{array} \right\} 3\delta\Delta z < 0, \qquad \text{(A.4.17)}$$

Equation (A.4.17) is strictly negative because both terms in the curvy brackets are strictly positive. This shows that the expected value to Country **C** of not being at war at time zero declines as the military effectiveness of its future opponents rises.

Nonetheless, we cannot conclude on this basis that scenario (a) is less sustainable when either Country **A** or **B** is militarily very superior to Country **C**. Quite the contrary: as we shall establish below, as those countries become more powerful militarily, it will be in the interest of Country **C** to defer a confrontation with either opponent because, by doing so, it will be able to ensure survival in period zero and face only one formidable opponent at time one. In other words, while equation (A.4.16) declines with increases in $\beta^B$, we shall demonstrate that the negative impact of an increase in $\beta^B$ on the expected value at time zero of Country **C** engaging Country **B** in period zero will be even larger. We address this scenario next.

**(b)** Country **B** and **C** engage in military conflict at $t = 0$ and the winner takes on Country **A** at $t = 1$. This case is identical to the previous one with the exception of Country **B** confronting Country **C** immediately instead of Country **A**.

$$\bar{\tau}_1 = \frac{\beta^v}{(1+\beta^v)^2} 3\Delta z \; ; \qquad v = \mathbf{B},\ \mathbf{C} \; . \qquad \text{(A.4.18)}$$

The expected net value of scenario (b) to countries **A** and $v$ at time 1 respectively are

$$V_1^A = \left[1 + \left(\frac{1}{1+\beta^v}\right)^2 3\Delta\right] z \qquad \text{(A.4.19)}$$

and

$$V_1^v = \left[2 + \left(\frac{\beta^v}{\beta^C+\beta^v}\right)^2 3\Delta\right] z \; . \qquad \text{(A.4.20)}$$

Equations (A.4.19) and (A.4.20) are analogous to equations (A.4.8) and (A.4.9) respectively: the expected value to Country **A** of engaging the survivor of the conflict between Country **B** and **C** at time one rises with $z$ but it falls with $\beta^v$, $v = \mathbf{B}, \mathbf{C}$. And the same properties hold for whichever of the two countries emerges victorious to face Country **A** in period one.

At time 0, when Country **B** and **C** face each other in conflict, we get $\bar{\tau}_0^C = \Omega\bar{\tau}_0^B$ where

$$\Psi \equiv \left[2 + \left(\frac{\beta^C}{1+\beta^C}\right)^2 3\Delta\right] \bigg/ \left[2 + \left(\frac{\beta^B}{1+\beta^B}\right)^2 3\Delta\right] > 1 \ . \tag{A.4.21}$$

Thus, we have

$$\bar{\tau}_0^B = \frac{\delta z \beta^B \beta^C \Psi}{(\beta^B + \beta^C \Psi)^2} \left[2 + \left(\frac{\beta^B}{1+\beta^B}\right)^2 3\Delta\right] \ , \tag{A.4.22}$$

and

$$\bar{\tau}_0^C = \frac{\delta z \beta^B \Psi}{(1 + \beta^B \Psi)^2} \left[2 + \left(\frac{\beta^C}{1+\beta^C}\right)^2 3\Delta\right] \ . \tag{A.4.23}$$

With (A.4.22) and (A.4.23), we can express the expected net values of scenario (b) to Country **B** and **C** at time 0 respectively as

$${}^bV_0^B = \left\{1 + \delta \left(\frac{\beta^B}{\beta^B + \beta^C\Psi}\right)^2 \left[2 + \left(\frac{\beta^B}{1+\beta^B}\right)^2 3\Delta\right]\right\} z \tag{A.4.24}$$

and

$${}^bV_0^C = \left\{1 + \delta \left(\frac{\beta^C\Psi}{\beta^B + \beta^C\Psi}\right) \left(\frac{(\beta^C - \beta^B)\Psi + 1}{\beta^B + \beta^C\Psi}\right) \left[2 + \left(\frac{\beta^C}{1+\beta^C}\right)^2 3\Delta\right]\right\} z \tag{A.4.25}$$

In line with the notation we adopted in case (a), the lowercase superscript $b$ to the left of the value function, $V$, now denotes the latter under case (b).For this equilibrium to be stable, Country **A** ought to find it optimal to decide not to attack Country **B** at $t = 0$. Since Country **B** and **C** are engaged in conflict at that time, Country **A** would find it optimal not to invest in its military at $t = 0$. Hence, the analog of (A.4.16) in this case is

$$ {}^{b}V_0^A = \left\{ 1 + \delta + \left[ \left( \frac{\beta^B}{\beta^B + \beta^C \Psi} \right) \left( \frac{1}{1 + \beta^B} \right)^2 + \left( \frac{\beta^C \Psi}{\beta^B + \beta^C \Psi} \right) \left( \frac{1}{1 + \beta^C} \right)^2 \right] 3\Delta\delta \right\} z \tag{A.4.26} $$

Note that

$$ \frac{\partial({}^{b}V_0^C)}{\partial \beta^B} = - \frac{(1 + \beta^C \Psi + \beta^C \Psi)}{(\beta^B + \beta^C \Psi)^3} \left[ 2 + \left( \frac{\beta^C}{1 + \beta^C} \right)^2 3\Delta \right] \delta z < 0, \tag{A.4.27} $$

which exceeds (A.4.17) in absolute value. Hence, the negative impact of an increase in $\beta^B$ on the expected value at time zero of Country **C** is larger if the latter is engaged in conflict with Country **B** in that period zero. And, as an extension, it is larger when, provided that it survives its confrontation with Country **B** in period zero, Country **C** would have to engage country **A** in military conflict in the next period.

**(c)** Finally, consider the scenario in which peace prevails indefinitely. It is not possible for all parties to invest no resources in military activities and for the peaceful equilibrium to be sustained because, in that case, one country could divert an infinitesimally small amount of resources to its military effort and invade and conquer its neighbor(s) without any resistance. Thus, peace can prevail as an equilibrium only if all countries allocate resources to military activities and neither chooses to attack its neighbor(s), similar in spirit to the non-appropriative equilibria with defensive fortifications described in Grossman and Kim (1995).

Consider the problem of Country **A** at $t = 0$. If country **A** arms in anticipation of engaging Country **B**, it will set its taxes at a level given by (A.4.12). Then, if Country **A** delays military action against **B** indefinitely, its indirectly utility will be given by

$$ {}^{c}V_0^A = \frac{z}{1 - \delta} \left\{ 1 - \frac{\delta \beta^B \Omega}{(1 + \beta^B \Omega)^2} \left[ 2 + \left( \frac{1}{1 + \beta^C} \right)^2 3\Delta \right] \right\} . \tag{A.4.28} $$

A similar argument holds for Country **B** at $t = 0$, which yields:

$$ {}^{c}V_0^B = \frac{z}{1-\delta}\left\{1 - \frac{\delta\beta^B\Omega}{(\beta^B + \beta^C\Omega)^2}\left[2 + \left(\frac{\beta^B}{\beta^B + \beta^C}\right)^2 3\Delta\right]\right\} . \qquad \text{(A.4.29)} $$

Country **C**, in contrast, would be getting $z$ ad infinitum, yielding an expected value of scenario (c) to it given by

$$ {}^{c}V_0^C = \frac{z}{1-\delta} . \qquad \text{(A.4.30)} $$

Scenario (c) could also apply if Country **A** stays on the sidelines, Country **B** and **C** arm to confront each other at time zero, but they delay military action indefinitely. In this case, we will get the following expected values for the three players:

$$ {}^{c}\hat{V}_0^A = \frac{z}{1-\delta} . \qquad \text{(A.4.28')} $$

A similar argument holds for Country **B** at $t = 0$:

$$ {}^{c}\hat{V}_0^B = \frac{z}{1-\delta}\left\{1 - \frac{\delta\beta^B\beta^C\Psi}{(\beta^B + \beta^C\Psi)^2}\left[2 + \left(\frac{\beta^B}{1 + \beta^B}\right)^2 3\Delta\right]\right\} . \qquad \text{(A.4.29')} $$

Country **C**, in contrast, would be getting $z$ ad infinitum, yielding an expected value of scenario (c) to it given by

$$ {}^{c}\hat{V}_0^C = \frac{z}{1-\delta}\left\{1 - \frac{\delta\beta^B\beta^C\Psi}{(\beta^B + \beta^C\Psi)^2}\left[2 + \left(\frac{\beta^C}{1 + \beta^C}\right)^2 3\Delta\right]\right\} . \qquad \text{(A.4.30')} $$

### 2.1.3 Sustainable Equilibria

We are now in position to assess which of the three equilibria could be sustained depending on parameter values. To start with, it is straightforward to establish that with sufficiently forward-looking rulers, for whom the discount factor $\delta$ is closer to one, case (c) yields the highest indirect utility. However, if the discount factor is relatively low, then either case (a) or case (b) would prevail over peace.

When this is the case, we will need to verify that a solution does exist; as I alluded to in the discussions of cases (a) and (b), it is possible that neither scenario is sustainable if it is not optimal for countries not in conflict in the first period to await the victor of an earlier conflict.

Keep in mind that Country **B** is in a precarious and unenviable position. If it comes under attack by either Country **A** or **C**, it has no choice but to engage in military conflict to defend itself. And for Country **B** to avoid a military conflict, both countries **A** and **C** need to find it in their interest to refrain from attacking Country **B**. Countries **A** and **C**, by contrast, are slightly better off because, as long as Country **B** does not initiate conflict, they can decide for themselves whether or not to engage Country **B** militarily.

Recalling that $\beta^A \equiv 1$, consider next the case in which Country **B** and **C** are evenly matched, i.e., $\beta^B = \beta^C > 1$. Under such parameter restrictions and substituting $\beta^B$ for $\beta^C$ in (A.4.16), (A.4.15) and (A.4.16) become

$$ {}^aV_0^2 = \left\{1 + \delta\left(\frac{\beta^2}{1+\beta^2}\right)\left(\frac{(\beta^2-1)\Omega+1}{1+\beta^2\Omega}\right)\left(2+\frac{3\Delta}{4}\right)\right\} z \qquad \text{(A.4.31)} $$

and

$$ {}^aV_0^3 = \left\{1 + \delta + \left[\left(\frac{1}{1+\beta^2\Omega}\right)\left(\frac{\beta^2}{1+\beta^2}\right)^2 + \frac{1}{4}\left(\frac{\beta^2\Omega}{1+\beta^2\Omega}\right)\right]3\Delta\delta\right\} z \qquad \text{(A.4.32)} $$

And equations (A.4.25) and (A.4.26) simplify to

$$ {}^bV_0^2 = \left\{1 + \frac{\delta}{2} + \frac{3\Delta\delta}{4}\left(\frac{\beta^2}{1+\beta^2}\right)^2\right\} z \qquad \text{(A.4.33)} $$

and

$$ {}^bV_0^3 = \left\{1 + \frac{\delta}{2} + \frac{3\Delta\delta}{4}\left(\frac{\beta^2}{1+\beta^2}\right)^2\right\} z \ . \qquad \text{(A.4.34)} $$

It is straightforward to verify that, $\forall\ \beta^2 = \beta^3 > 1$, equation (A.4.32) exceeds (A.4.34). Thus, Country **C** will prefer to defer a confrontation early on. Moreover, $\exists\ \beta^2 = \beta^3 > 1$ such that (A.4.31) is greater than (A.4.33) and Country **B** prefers to engage Country **A** immediately.

Here is the reason why: Equation (A.4.32) evaluated at $\beta^2 \to 1$ equals 1 + $\delta$ + $3\delta\Delta/4$ and (A.4.34) evaluated at $\beta^2 \to 1$ equals 1 + $\delta/2$ + $3\delta\Delta/16$. Hence, in the limit when $\beta^2 \to 1$, (A.4.32) strictly exceeds (A.4.34). Equation (A.4.32) evaluated at $\beta^2 \to \infty$ equals 1 + $\delta$ + $3\delta\Delta/4$ and (A.4.34) evaluated at $\beta^2 \to \infty$ equals 1 + $\delta/2$ + $3\delta\Delta/4$ . As a result, in the limit when $\beta^2 \to \infty$, (A.4.32) strictly exceeds (A.4.34). Note that the net expected values of scenarios (a) and (b), ${}^aV_0^2$, ${}^aV_0^3$, ${}^bV_0^2$, ${}^bV_0^3$, are strictly monotonic in $\beta^2$. This establishes that, $\forall$ $\beta^2 = \beta^3 > 1$, ${}^aV_0^3 > {}^bV_0^3$.

In similar fashion, we can evaluate equation (A.4.31) at $\beta^2 \to 1$ and get 1 + $\delta/2$ + $3\delta\Delta/16$. And we can evaluate (A.4.33) at $\beta^2 \to 1$ to generate 1 + $\delta/2$ + $3\delta\Delta/16$. Equation (A.4.31) evaluated at $\beta^2 \to \infty$ yields 1 + $2\delta$ + $3\delta\Delta/4$ and (A.4.33) evaluated at the same point generates 1 + $\delta/2$ + $3\delta\Delta$/A.4. Given that the net expected values of scenarios (a) and (b), ${}^aV_0^2$, ${}^aV_0^3$, ${}^bV_0^2$, ${}^bV_0^3$, are strictly monotonic in $\beta^2$, it follows that $\forall$ $\beta^2 = \beta^3 \in [1, \infty]$, (A.4.31) exceeds (A.4.33). That is, ${}^aV_0^2 > {}^bV_0^2$

Given these findings, we conclude that, $\forall$ $\beta^2 = \beta^3 > 1$ case (a) will be the stable equilibrium.

Next consider parameter values $\beta^3 > \beta^2 = 1$ such that Country **C** dominates the other two countries in military technology. Rewriting (A.4.14) under the assumption that $\beta^2 = 1$, we get

$$ {}^aV_0^1 = \left\{ 1 + \frac{\delta}{2} + \frac{3\Delta\delta}{4} \left( \frac{1}{1+\beta^3} \right)^2 + \right\} z \qquad \text{(A.4.35)} $$

And rewriting (A.4.16) with $\beta^2 = 1$ yields

$$ {}^aV_0^3 = \left\{ 1 + \delta + \frac{3\Delta\delta}{4} \left( \frac{\beta^3}{1+\beta^3} \right)^2 + \right\} z \qquad \text{(A.4.36)} $$

Going through the same steps with equations (A.4.24) and (A.4.25), we generate

$$ {}^bV_0^1 = \left\{ 1 + \delta + \left[ \frac{1}{4} \left( \frac{1}{1+\beta^3\Omega} \right) + \left( \frac{\beta^3\Omega}{\beta^2+\beta^3\Omega} \right) \left( \frac{1}{1+\beta^3\Omega} \right)^2 \right] 3\Delta\delta \right\} z \qquad \text{(A.4.37)} $$

$$
{}^{b}V_0^3 = \left\{ 1 + \frac{\delta}{2} \left( \frac{(\beta^3 - 1)\Omega + 1}{1 + \beta^3\Omega} \right) \left[ 2 + \left( \frac{\beta^3}{1 + \beta^3} \right)^2 \right] 3\Delta \right\} z \; . \qquad \text{(A.4.38)}
$$

It is straightforward to verify that, $\forall\ \beta^3 = 1$, equation (A.4.37) exceeds (A.4.35). Thus, Country **A** will prefer not to engage Country **B** in the first period. In contrast, Country **C** will want to engage Country **B** in the first period if $\beta^3$ is sufficiently large because, $\exists\ \beta^3 > 1$ such that (A.4.38) is greater than (A.4.36).

To demonstrate that this is the case, we can proceed as we did above: Equation (A.4.35) evaluated at $\beta^3 \to 1$ equals $1 + \delta/2 + 3\delta\Delta/16$ and (A.4.37) evaluated at $\beta^3 \to 1$ equals $1 + \delta + 3\delta\Delta/\text{A.4}$. Equation (A.4.35) evaluated at $\beta^3 \to \infty$ equals $1 + \delta/2$ and (A.4.37) evaluated at $\beta^3 \to \infty$ equals $1 + \delta$. Again, due to the fact that the expected payoffs ${}^{a}V_0^1$, ${}^{1}V_0^3$, ${}^{b}V_0^1$, ${}^{b}V_0^3$ are strictly monotonic in $\beta^3$, we can conclude that, $\forall\ \beta^3 \in [1,\, \infty] \wedge \beta^3 > \beta^2 = 1$, ${}^{b}V_0^1 > {}^{a}V_0^1$.

Now take equations (A.4.36) and (A.4.38): (A.4.36) evaluated at $\beta^3 \to 1$ yields $1 + \delta + 3\delta\Delta/16$ and (A.4.38) evaluated at the same point generates $1 + \delta/2 + 3\delta\Delta/16$. Equation (A.4.36) evaluated at $\beta^3 \to \infty$ yields $1 + \delta + 3\delta\Delta/4$ and (A.4.38) evaluated at the same point generates $1 + \delta + 3\delta\Delta/2$. Given that ${}^{a}V_0^1$, ${}^{a}V_0^3$, ${}^{b}V_0^1$ and ${}^{b}V_0^3$ are strictly monotonic in $\beta^3$, it follows that $\exists\ \beta^3 \in (1, \infty)$ for which (A.4.38) exceeds (A.4.36).

Thus, we conclude that, $\exists\ \beta^3 > \beta^2 = 1$, case (b) will be the stable equilibrium.

In terms of the advent of Abrahamic monotheisms, one can think of the role of religious differences and affinities as coming to bear on the $\beta$'s in this model. Specifically and in line with Chapters 2 and 3 as well as the discussion in Section A.4.2 above, we can conjecture that a monotheist country bordering one with a non-monotheist religious creed was tantamount to the former having a considerably higher $\beta$ vis-a-vis the latter. We have already established above that such a scenario would make the monotheist country allocate relatively more of its resources to its military, as a result of which its likelihood of triumphing over its neighbor in a confrontation would be relatively higher.

By contrast, two bordering countries with their majorities subscribing to different monotheisms defines a situation analogous to both countries having relatively high $\beta$'s. We have already seen that such countries would allocate relatively more resources to military conflict, although their likelihood of prevailing over their monotheistic adversaries would not be that much higher because both countries would have similar $\beta$'s.